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and Peak-Hold Methods of Flutter Onset  
Prediction**

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# SOME OBSERVATIONS ON THE HOUBOLT-RAINEY AND PEAK-HOLD METHODS OF FLUTTER ONSET PREDICTION

By

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## INTRODUCTION

In a paper presented at a flight flutter testing symposium in 1958 Houbolt and Rainey<sup>1</sup> proposed a subcritical response method for flutter onset prediction that can be used either with sinusoidally varying forced excitation (shaker) or randomly varying forced excitation (turbulence). The method is suitable for both flight and wind-tunnel flutter testing. The analytical foundation of this method is straight forward and simply requires the measurement of the amplitude of the response in the structural mode important to flutter. The reciprocal of these amplitude measurements are plotted against a flow parameter such as density as flutter is being approached and the resulting curve extrapolated to the flutter condition which occurs when the amplitude reciprocal equals zero. The method predicts a linear trend of amplitude reciprocal with flow parameter. Seventeen years later in their summary report of an active flutter suppression study, Sandford, Abel, and Gray<sup>2</sup> presented some results using a subcritical response method they called the Peak-Hold Method. They noted that the Peak-Hold Method is similar to the Houbolt-Rainey Method but did not offer any elaboration. As far as can be determined no one has made an explicit connection between the two methods, even though the Peak-Hold Method has become a standard method used during flutter testing in the NASA Langley Transonic Dynamics Tunnel<sup>3</sup> which is a major wind-tunnel facility in the United States for aeroelastic testing.<sup>4</sup>

The purpose of the present paper is to examine the relationship between these two methods. Although both methods can be applied to systems that are being either excited by sinusoidally varying forces such as those produced by a shaker or by randomly varying forces such as those produced by flow turbulence, the present discussion will focus on random excitation. There is no loss of generalization by focusing on random excitation. In particular, the physical system that will be considered in discussing the two methods is a wind-tunnel flutter-model wing that is being excited only by wind-tunnel turbulence. Neither method requires knowledge of the magnitude of the exciting force. Both methods require the determination of the response of the system, amplitude of response in the mode important to flutter, but the exact values of the response are not required by either method. Consistent measurements of quantities proportional to the amplitude are all that are needed. Furthermore, neither method can be used to determine the damping of the wing.

## HOUBOLT-RAINEY (H-R) METHOD

The analytical development of the Houbolt-Rainey Method<sup>1</sup>, hereafter referred to as the H-R Method, is based on an assumed mode solution of the system of differential equations that describe the

flutter behavior of a wing that is excited by external forces such as a shaker or turbulence. (What is called turbulence here is referred to as gusts in ref. 1.) They developed a relatively simple expression for the amplitude of vibration of the surface in the structural mode important to flutter. This expression with both shaker and turbulence forces present is

$$a = \frac{Q_s + \rho Q_t}{V_f^2 A_f (\rho - \rho_f)} \quad (1)$$

where  $Q$  is a generalized force,  $A$  is a generalized aerodynamic force that is a function of Mach number and reduced frequency,  $V$  is the fluid velocity,  $\rho$  is the fluid density, and the subscripts  $f$ ,  $s$ , and  $t$  refer to flutter, shaker, and turbulence, respectively.

By inverting eq. 1 and separating the shaker and turbulence terms the following relationships are obtained where the vertical bars on either side of a symbol are used to denote the magnitude of the variable.

$$\frac{1}{|a|} = \frac{V_f^2 |A_f|}{|Q_s|} (\rho_f - \rho) \quad \text{Shaker only} \quad (2)$$

$$\frac{1}{|a|} = \frac{\rho_f V_f^2 |A_f|}{|Q_t|} (1/\rho - 1/\rho_f) \quad \text{Turbulence only} \quad (3)$$

Note that for the shaker only case (eq. 2) the amplitude is predicted to vary linearly with density  $\rho$  as flutter is approached, whereas for the turbulence only case (eq. 3) the amplitude is predicted to vary linearly with the reciprocal of the density. At flutter the magnitude of the amplitude  $|a|$  would be expected to be very large so the term  $\frac{1}{|a|}$  would approach zero. The relationships developed in ref.1 assumed that the flow velocity is held constant while the density is varied as flutter is being approached. Relationships similar to eqs. 2 and 3 hold, however, if both the velocity and density are varying simultaneously, that is, the dynamic pressure  $q = \frac{1}{2}\rho V^2$  is being changed. In many instances this would likely be the case so it would be convenient to examine the variations of  $\frac{1}{|a|}$  with the dynamic pressure (shaker case) or the reciprocal of the dynamic pressure (turbulence case). Of course, when  $q$  is being varied the trends may not be exactly linear because the Mach number and the reduced frequency are varying. In the neighborhood of flutter, however, it would be expected that the trends would be nearly linear. Furthermore, it should be noted that the presence of significant buffeting flows over the wing may also cause the trend to deviate from being linear.

These relationships, eqs. 1 and 2, provide a simple means to predict the flutter condition from subcritical response measurements made during experiments when the wing is either excited by a shaker or by flow turbulence. By measuring the dynamic response of the surface in the structural mode important to flutter at several different subcritical flow conditions and then plotting the reciprocal of these amplitude measurements versus the appropriate flow quantity, it is possible to extrapolate to the flutter condition which is the flow condition at which the amplitude reciprocal becomes zero.

Some experimental results from applying this method are presented in ref. 1, and are repeated here in figure 1. Results are shown for a variety of wing configurations at subsonic and transonic speeds. For these cases the method did accurately predict the flutter condition as long as the subcritical response measurements were made within 20 percent of the flutter dynamic pressure. The amplitude values needed by Houbolt and Rainey to apply the turbulence technique were determined by using auto-

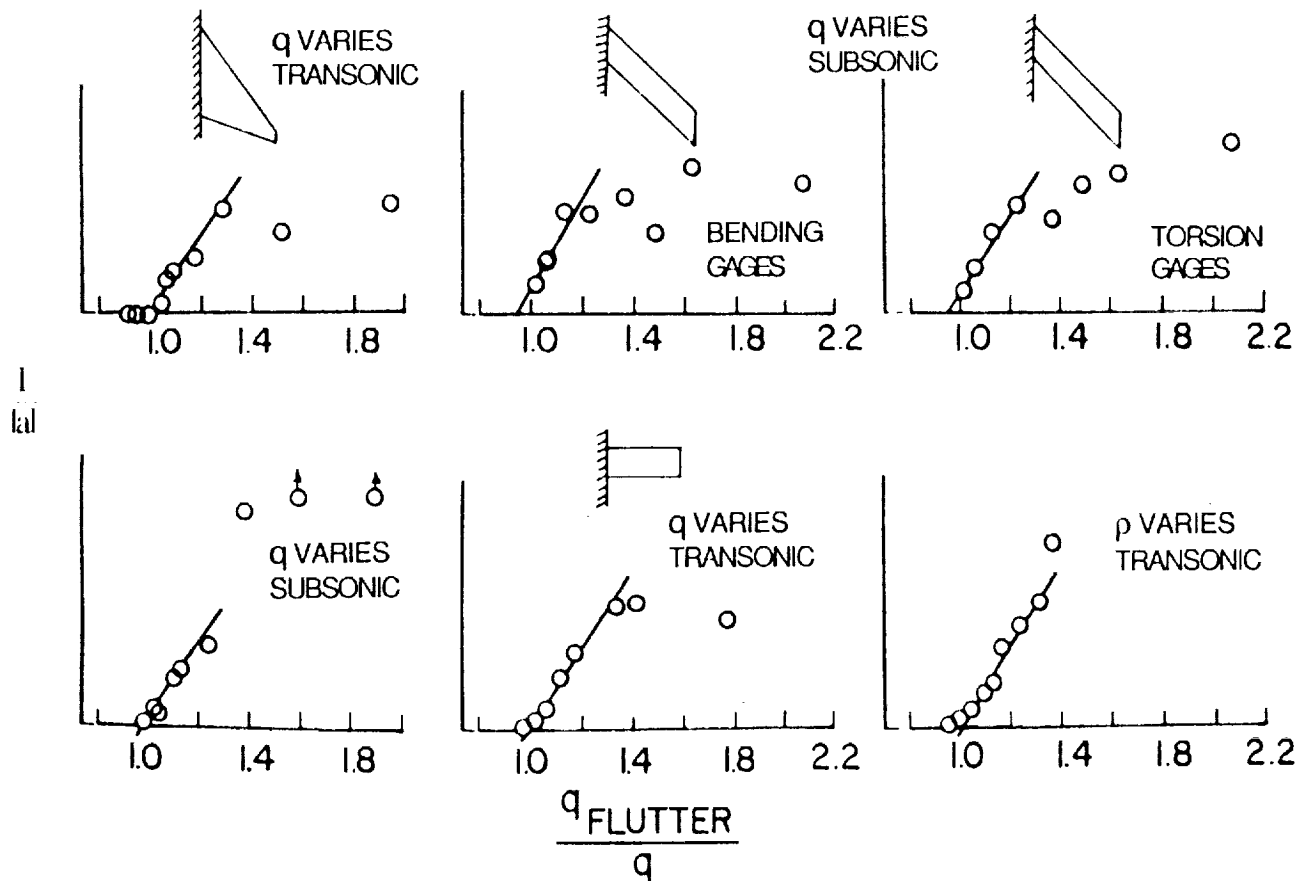


Figure 1. - Extrapolation to flutter onset condition using Houbolt-Rainey Method. (Ref. 1.)

spectra (power spectral densities) of the response measured during the wind-tunnel tests. Because the autospectra units were amplitude squared/Hertz, they had to convert, in effect, each measured spectrum into an amplitude spectrum to determine the amplitude in the mode important to flutter. An amplitude autospectrum is a spectrum for which the value of the spectrum at each respective frequency represents

the mean square value of the response at that frequency. The units of this spectrum are simply amplitude units squared, say, for example, in.<sup>2</sup> The response amplitudes determined in this way are what were used in ref. 1 to obtain the results presented in figure 1.

### PEAK-HOLD METHOD

By the early 1970's it had become accepted practice during flutter testing in the Transonic Dynamics Tunnel to use frequency analyzers to track response frequencies of models during the approach to flutter. Apparently Sandford had observed that there appeared to be a trend associated with the amplitude of the spectral peaks, so his intuition led him to try what has become to be known as the Peak-Hold Method. (The peak-hold function is built into most off-the-shelf transfer function analyzers.) Following Sandford's lead other researchers have also used this method with varying degrees of success.<sup>3,5</sup> Of course, no subcritical response flutter onset prediction method has yet been developed that works accurately in all situations.

The basic measurement needed for applying this method is, like for the H-R Method, the amplitude of the structural mode that is important to flutter. In this case, however, the needed amplitude information is obtained by using what is called a peak-hold spectrum. Although a peak-hold autospectrum (amplitude squared/Hertz) may be used, it is usually more convenient to use a peak-hold amplitude spectrum, that is, a spectrum for which the ordinate values are the mean squares values of the response amplitude. The use of an amplitude spectrum is appropriate because the response is limited to a few discrete frequencies and is not broad band random noise. Next we will discuss some notions concerning spectra, in particular, amplitude spectra and peak-hold spectra. The discussion is by no means elegant, and is somewhat "schematic" in nature, but it should prove useful to readers who are not familiar with spectral analysis. (There are many fine books available on time series analysis of random data. See, for example, ref. 6.)

To aid in understanding a peak-hold spectrum let us consider a digital-signal analyzer with an analysis bandwidth of  $N(\Delta f)$  where  $N$  is an integer constant and  $\Delta f$  is the frequency resolution. Typically  $N$  would be of the order of several hundred. If, for example,  $N=200$  and the frequency bandwidth of interest was from zero to 50 Hertz, then the frequency resolution  $\Delta f$  would be 0.25 Hertz. The spectrum is determined at  $N$  values of frequency, each value of frequency separated from adjacent values of frequency by an amount  $\Delta f$ . The signal is in effect passed through a series of  $N$  "windows," or filters, that are  $\Delta f$  wide and centered at the frequencies  $f_N$ . The analyzer determines the mean square value of the signal in each frequency window. The amplitude spectrum is the foundation element of the peak-hold spectrum.

The process of determining a peak-hold spectrum proceeds as follows: An initial amplitude spectrum of the wing response is calculated and its values stored in the analyzer memory. This spectrum and subsequent ones are obtained from very short time segments of the response so the spectrum

represents the variation of the response amplitude with frequency at an "instant" of time rather than providing statistical information representing the overall random process. Each frequency window has a unique location in analyzer memory. In addition the spectrum is displayed on an oscilloscope screen for visual monitoring. A second amplitude spectrum is then determined and its values compared with the initial spectrum stored in memory. The memory is updated at each frequency window for which the value of the new spectrum is larger than the value stored in memory. This process of determining spectra and comparing new values with stored values and displaying the results is repeated until the spectrum stored in memory is not being changed as determined by visual observation of the display on the oscilloscope screen. Experience has shown that the oscilloscope screen will typically show that the spectrum changes rapidly at the beginning of the process (perhaps the first 30 seconds or so) and then appears to remain unchanged as time passes. Once the spectrum has been observed to converge, the spectrum calculation-comparing process is stopped. The spectrum that is stored in memory now is the peak-hold spectrum and represents the maximum mean square amplitude of the wing response at each of the N frequencies that occurred during the time that the spectra were being determined. These peak values are held in memory throughout this process, hence the name, peak-hold.

In applying the Peak-Hold Method to flutter onset prediction this process is repeated at several different flow conditions. It has become standard practice to plot the reciprocal of the amplitude of the structural mode important to flutter versus dynamic pressure and then extrapolate this curve to predict the flutter onset condition. The H-R Method suggests that examining the  $\frac{1}{|a|}$  trend with the reciprocal of dynamic pressure would be a better choice because the predicted trend is linear. If the trend is indeed linear with the reciprocal of the dynamic pressure, then the trend is concave with dynamic pressure. Obviously, it is easier to extrapolate a linear trend than it is to extrapolate a non-linear trend. This is illustrated by the data presented in figures 2 and 3. The data in these figures were obtained during the flutter tests described in ref. 7. The model was a 45°-sweep delta wing. The measured flutter dynamic pressure for this model was about 46 psf. Presented in figure 2 is the variation with dynamic pressure of the measured reciprocal of the response amplitude in the mode that was important to flutter. The response amplitude is the square root of the value of the peak in the spectrum that occurs at the frequency of the structural model important to flutter. Therefore,  $|a|$  is the root mean square of the amplitude. These data have been fitted with two curves, a linear least squares fit and a second degree polynomial fit. The second degree curve fits the data better as would be expected if the trend of the amplitude is indeed linear with the reciprocal of  $q$ . The linear fit, however, gave a projection of the flutter dynamic pressure, 47 psf, that is closer to the experimental value, 46 psf. An important point to be made here is that it is obvious that small changes in the second degree fit could make large changes in the flutter onset prediction. Presented in figure 3 is the variation of the same amplitude reciprocal with the reciprocal of the dynamic pressure. In this case the linear curve provides an excellent fit to the

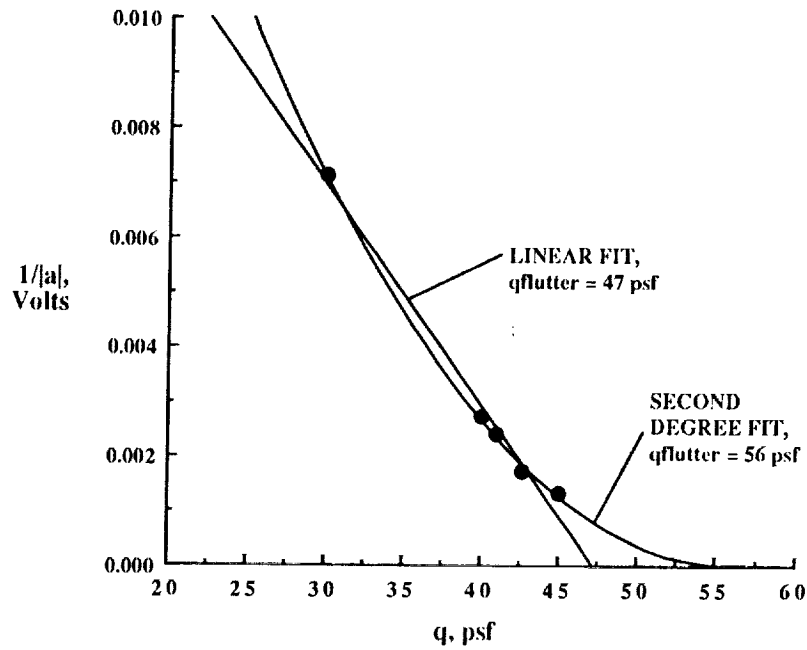


Figure 2. - Variation of the reciprocal of response amplitude with dynamic pressure.

data. The predicted flutter dynamic pressure, 51 psf, is about ten percent higher than the experimental value, 46 psf. The second degree polynomial fit shown in this figure is almost the same as the linear fit. Data of the type shown in figures 2 and 3, which are not untypical of wind-tunnel model subcritical flutter results, indicate that it is easier to extrapolate the linear curve in figure 3 than it is to extrapolate

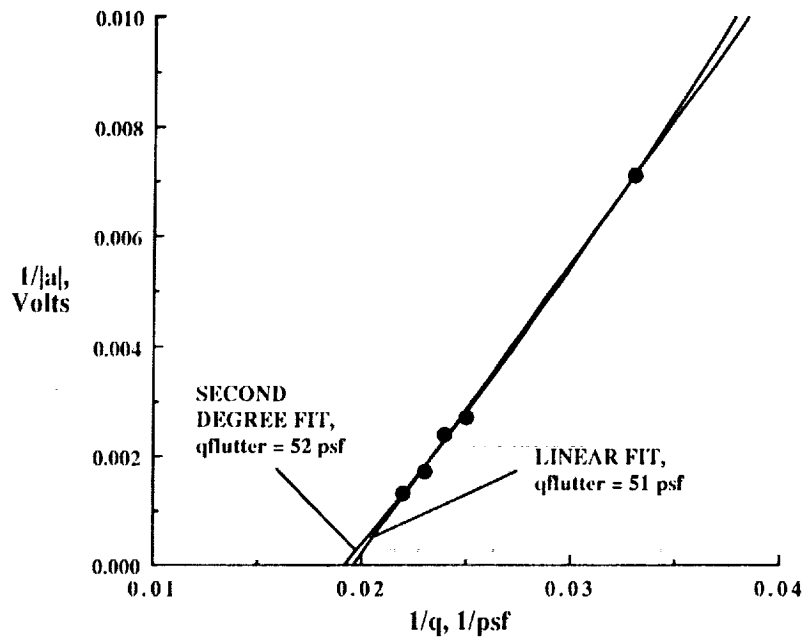


Figure 3. - Variation of the reciprocal of response amplitude with the reciprocal of dynamic pressure.



the second degree curve in figure 2. It is suggested, therefore, that in future applications amplitude trends with the reciprocal of the dynamic pressure be used rather than trends with dynamic pressure.

### COMPARISON OF H-R AND PEAK-HOLD METHODS

It appears from the proceeding discussion that the only difference between the H-R Method and the Peak-Hold Method is in the spectrum used to determine the amplitude. Houbolt and Rainey used a classical autospectrum (power spectral density in amplitude squared/Hertz determined by analog means) because that was the only capability that was available to them at the time they did their work. To obtain the needed amplitude data they, in effect, converted the classical autospectra to an amplitude spectra. (As a junior researcher who helped acquire some of the experimental data presented in ref. 1, the author has personal knowledge that this was the case.) If they were doing their work today, they would undoubtedly use amplitude spectra determined by digital means. One could argue, however, that they might not choose to use peak-hold spectra. So the question of difference in spectra still exists. The following discussion addresses this question.

For the sake of argument let us assume that the turbulence that is exciting the wing is a linear, ergodic random process with zero mean value and a "nearly" normal probability distribution. A normal probability distribution is shown in figure 4. This distribution is presented in terms of a variable which

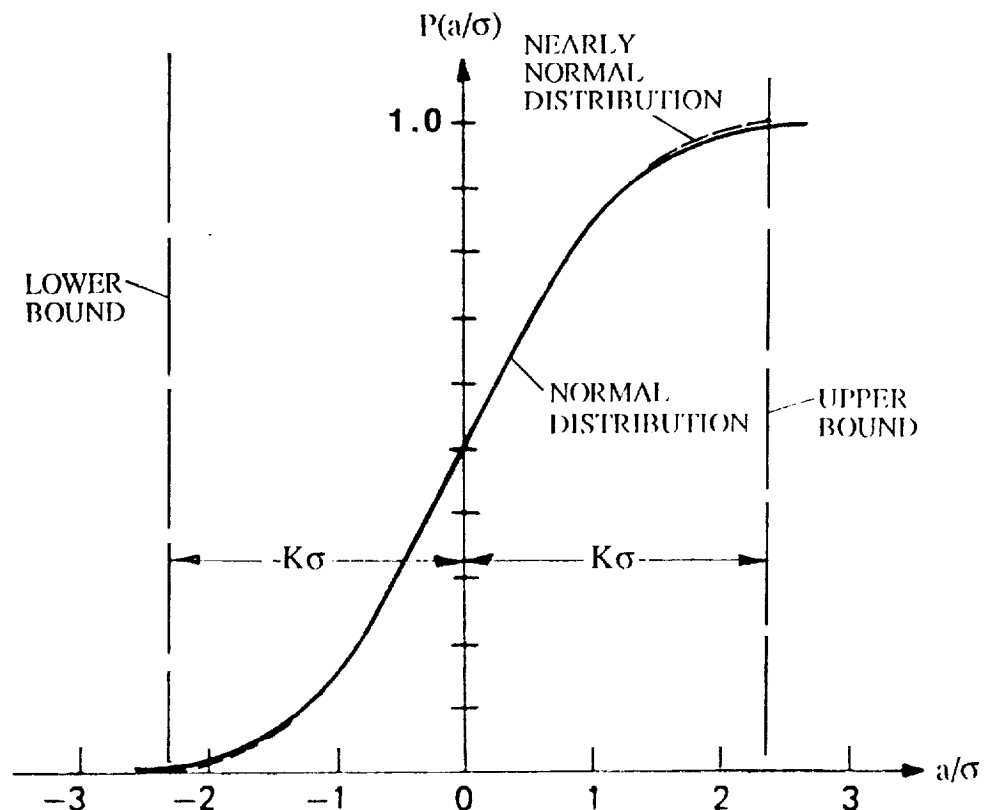


Figure 4. - Normal probability distribution function.

is the ratio of the response  $a$  to the standard deviation  $\sigma$ . These assumptions may appear to be restrictive, but they are in general representative of such physical processes. The notion of a "nearly" normal probability distribution, however, is quite important to the following arguments so some clarification is needed. Theoretically a process that has a normal probability distribution has a finite probability of exceeding any selected upper limit, no matter how large, and a finite probability of being less than any selected lower limit, no matter how small. Natural processes, however, are generally bounded. The dashed vertical lines in the figure are used to indicate upper and lower bounds. The dashed modifications to the normal probability distribution indicate how this function might change near these bounds. The extreme values would be  $K\sigma$  and  $-K\sigma$  where  $K$  is a constant that is characteristics of the particular random process. There are a variety of reason why physical systems have limiting values. Some of these are nonlinear effects such as springs that harden when the deflection is increased beyond a certain value, pressures that cannot be lower than an absolute vacuum, work done that cannot exceed the available energy, and motion of particles inside a container that cannot exceed the boundaries of the vessel. Although bounded, many physical processes still exhibit characteristics of normal processes between their upper and lower limits. So, a process that has a "nearly" normal probability distribution function is a process that generally exhibits many of the characteristics attributed to a normal process except that it is a process that has both upper and lower limits. It is generally accepted that wind-tunnel turbulence is a process that exhibits such characteristics.

If one were to determine an amplitude autospectrum of the response of the wing to wind-tunnel turbulence over a sufficiently long period of time (typically 30 to 60 seconds in most applications), a spectrum would be determined wherein each point in the spectrum is equal to the mean square value of the response at each of the  $N$  frequencies. If this time is sufficiently long, then the spectrum represents the characteristics of the complete random process and not just what is going on at an instant of time. Thus, if the spectrum had a peak at frequency  $f_p$ , the value of the spectrum at  $f_p$  would be equal to the mean square amplitude at that frequency. Here this value would be equal to the square of the standard deviation  $\sigma$  at  $f_p$  because the mean square value and the square of the standard deviation are the same quantity for a process with zero mean value. It is important to understand that the standard deviation  $\sigma$  of that portion of the random response occurring at frequency  $f_p$  is not the standard deviation of the total random process. If we were to determine a peak-hold amplitude spectrum, then a similar looking spectrum would result. The amplitude autospectrum and the peak-hold spectrum would have peak values of amplitudes occurring at the same frequencies, but the magnitude of these amplitudes would be different because the amplitudes of the peak-hold spectrum are maximum mean square values that occur during the time that the spectra were being determined whereas the amplitudes of the autospectrum are mean-square values representative of the entire random process. If the process did indeed have a normal probability distribution function then it would be probable for both extremely large and extremely small values to exist and, therefore, the peak-hold spectra would not converge as it has been observed

to do. Indeed, it could not converge. Because of the upper and lower bounds that are exhibited by physical systems and because the peak-hold spectrum process is conducted over a relatively long time, it is highly probable that amplitude responses of the order of the extreme values will occur. Therefore, the maximum response as obtained by the peak-hold spectra may be thought of as  $K^2\sigma^2$  values, that is, mean square values that occur when the amplitude of response is near the extreme values. Thus, the magnitudes of amplitude spectrum, which are  $\sigma^2$  values, are related to the magnitudes of the peak-hold spectrum, which are  $K^2\sigma^2$  values, by the constant multiplier  $K^2$ . Because in applying either method the absolute values of the response are not important, then there no need to know the value of  $K$ . Therefore, there should be no difference in the results no matter which spectrum is used.

It is now clear that the Peak-Hold Method and the H-R Method are the same method. Because there is an analytical foundation for the H-R Method, then there is an analytical foundation for the Peak-Hold Method, verifying that Sandford's insight was very astute. So, the fact that the Peak-Hold Method has proven to be a useful subcritical response techniques for flutter onset prediction is not just fortuitous, it should be expected to be so, although like all subcritical methods, it will not be reliable in all applications. Indeed, the Peak-Hold Method is not actually a "method." It is more precisely a particular means by which the data required to use the H-R Method are obtained. In recognition of that fact it seems appropriate that in the future the designation Peak-Hold Method should be replaced with the more appropriate designation Houbolt-Rainey Method.

#### CONCLUDING REMARKS

A subcritical response method for flutter onset prediction developed by Houbolt and Rainey in 1958 has been compared with the Peak-Hold Method which was apparently first applied to flutter onset prediction by Sandford, Abel, and Gray in the early 1970's. The rational argument presented shows that the two methods are not different methods, but are actually the same method. So, because there is an analytical foundation for the Houbolt-Rainey Method, then there is the same analytical foundation for the Peak-Hold Method. Therefore, it is not just fortuitous that the Peak-hold Method has proven to be a useful tool in flutter onset prediction.

Further, it is suggested that, in applying the Peak-Hold Method in cases where turbulence is used as the excitation force, the variation of the reciprocal of the response amplitude with the reciprocal of the dynamic pressure be used to extrapolate to flutter onset rather than the variation with dynamic pressure which is current practice because the linear trend which is predicted to occur for the former case is easier to extrapolate to the flutter condition than the nonlinear trend predicted to occur for the latter case. Finally, because the method is actually the Houbolt-Rainey Method, the Peak-Hold Method being only a means by which the data needed to apply the Houbolt-Rainey Method are acquired, it is suggested that the method be referred to in the future as the Houbolt-Rainey Method.

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